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FIRST-ORDER "INTERRUPTABLE" DESIGNS

by

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ABSTRACT

Even a well-designed experiment interrupted before its intended completion may prove inefficient, posing serious data analysis problems and leaving important questions unanswered. This report addresses the construction of "interruptable" designs for those cases where a first-order model is assumed and factors at only two levels are considered. The maximum $|\underline{X}'\underline{X}|$ criterion was used in developing "interruptable" designs for two different design strategies which attempt to limit the information lost if an experiment is prematurely ended. The resulting designs for four to nine factors are evaluated and selection of an "interruptable" design is discussed.

Key Words

Interruptable Designs

First-order Model

Two-level Designs

$|\underline{X}'\underline{X}|$ Criterion

I. INTRODUCTION

An important element of most research investigations is a well-designed experiment. Unfortunately, there exist situations in which an experiment may be unavoidably interrupted (and ended) before it is completed. Most applied statisticians have, no doubt, encountered such situations, which seem to be primarily caused by equipment failures, unanticipated budget changes, or revised operational requirements.

Of course, even a well-designed experiment interrupted before its intended completion may prove inefficient, posing serious data analysis problems and leaving important questions unanswered. This report addresses the construction of "interruptable" designs for those cases where a first-order model is assumed, factors at only two levels are considered, and experimental runs are at a premium. These designs limit the adverse effects of an incomplete experimental program.

The research described in this report was sparked by some Navy problems in which the experimenter did not know the total number of runs available, although he was guaranteed a minimum number. This situation occurs particularly when experiments are conducted at sea. For example, an experiment aboard a submerged submarine might be cut short if the submarine had to surface unexpectedly.

II. PROBLEM DISCUSSION

Before a judgment about the goodness of a design may be made, a criterion must be selected. In this report, the $|\underline{X}'\underline{X}|$ criterion has been adopted. This criterion, which has been widely used for developing optimal designs [e.g., Box and Draper (1971), Mitchell (1972, 1974a), St. John and Draper (1975)], is independent of scale and simplifies when augmenting existing designs.

As mentioned in the previous section, the interruptable designs discussed in this report are appropriate when the number of runs that may be completed is uncertain. In practice there may be situations where this number remains unknown until immediately preceding experimentation. In such cases interruptable designs are not necessary, although the experimenter must have on hand a design consisting of the appropriate number of runs. To assure this, a series of designs covering the range of runs possible should be available.

Many designs which are optimal for various numbers of factors and runs are available in the literature [e.g., Webb (1971) and Mitchell (1972)]. Mitchell (1974b) has also developed a computer program (DETMAX) for constructing designs with various numbers of runs. For example, suppose an experiment were to investigate eight factors each at two levels, and it were known that there would be a total of nine, ten or eleven runs available. If the experimenter would be informed of the exact number of runs immediately prior to the first run, he or she could have ready the three designs (listed in Figure 1), which have been constructed by the

Run No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	1	-1	1	1	1	1	1	-1
2	1	1	-1	1	1	-1	-1	-1
3	-1	1	-1	1	-1	1	1	-1
4	1	-1	-1	-1	-1	-1	1	1
5	-1	1	1	-1	1	-1	1	-1
6	-1	-1	1	1	-1	-1	-1	1
7	-1	-1	-1	-1	1	1	-1	-1
8	1	1	1	-1	-1	1	-1	1
9	-1	1	-1	1	1	1	1	1

Run No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	-1	1	-1	-1	1	-1	-1	-1
2	-1	-1	-1	1	-1	-1	1	1
3	1	-1	1	-1	-1	-1	-1	-1
4	1	-1	-1	1	1	1	-1	1
5	1	1	1	1	1	-1	1	-1
6	1	1	-1	1	-1	1	-1	-1
7	-1	-1	1	1	1	1	-1	-1
8	1	-1	-1	-1	1	1	1	-1
9	-1	1	1	-1	-1	1	1	1
10	1	1	1	-1	1	-1	-1	1

Run No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	-1	-1	-1	-1	-1	1	1	1
2	1	1	-1	-1	-1	-1	1	-1
3	1	-1	1	-1	1	-1	1	1
4	-1	-1	-1	1	1	-1	-1	-1
5	1	1	1	1	1	1	1	-1
6	1	-1	1	-1	-1	1	-1	-1
7	-1	1	-1	-1	1	1	-1	1
8	-1	1	1	1	-1	-1	-1	1
9	1	-1	-1	1	-1	1	-1	1
10	-1	-1	1	1	-1	1	1	-1
11	1	1	-1	1	-1	-1	1	1

Figure 1: Designs For 8 Factors in 9, 10 or 11 Runs

DETMAX program [Mitchell (1974b)].

Interruptable designs, as defined in this report, are intended for use in situations where the number of runs will not be known until the experiment is under way. Quite often this is not until the last possible run has been completed. Of course, the experimenter in selecting a design could ignore the possibility of interruption and hope that enough runs would be available to complete the selected design.

In light of the assumptions in the previous section, it is reasonable that the experimenter adopting this view would select a Plackett-Burman design of minimum size. This design is not only parsimonious with the number of runs, but also provides the maximum achievable $|\underline{X}'\underline{X}|$ for the given number of runs and results in uncorrelated parameter estimates. However, an unwarranted assumption that the design will be completed may prove dangerous, for if it is not completed, valuable information may be lost. The interruptable designs discussed in this report are constructed to limit the adverse effects of an incomplete experimental program.

The degree to which the number of runs can be predicted varies with experimental setting. In some cases the experimenter is confident that most of the runs can be completed, whereas in other cases even the first few runs are questionable. Therefore, interruptable designs to cover this wide range of experimental settings are necessary.

For purposes of this report, the following two types of interruptable experiments will be defined:

(1) An experiment where no runs are guaranteed.

(2) An experiment where an initial block of runs is guaranteed but runs after that are uncertain.

The first type of experiment requires a design which accounts for interruption at any point. Webb (1968) proposed one-at-a-time designs for such situations. These designs allow the factors to be introduced one-at-a-time in the presumed order of importance. Such designs have been criticized because of their poor variance properties. However, they are valuable as interruptable designs in that the effects of some of the factors can be estimated regardless of when the experiment is interrupted. In addition, Daniel (1958) has pointed out that the properties of these designs can be greatly improved with the completion of several additional experimental runs.

The research described in this report was directed at identifying designs appropriate for the second type of interruptable experiment. To put bounds on the problem scope, it will be assumed that enough runs are guaranteed to allow for an estimate of each factor effect. That is, for the case of k factors it will be assumed that if interruption occurs, it will not occur before the k+1st run. It will be further assumed that, because runs are at a premium, the maximum number of runs for consideration is that necessary to complete the smallest Plackett-Burman design for k factors.

Within these limitations , one strategy for developing an interruptable design is to begin with an optimal k+1 run design and augment it run by run to produce the best possible design at each stage. An alternate strategy is to use the Plackett-Burman design, which is optimal for the completed experiment, and order the runs to provide the best possible k+1, k+2,... run designs.

The first strategy provides an optimal design if the experiment is interrupted after k+1 runs, but it does not necessarily provide an optimal design for experiments with more than k+1 runs. The second strategy provides an optimal design for the completed experiment but it does not necessarily provide an optimal design for interrupted experiments. Thus, neither strategy leads to an optimal design for each possible point of interruption. However, each strategy does attempt to limit the information lost if the experiment is interrupted at one of the nonoptimal stages.

This report examines the performance of these two strategies in those cases involving four to nine factors. It is implicitly assumed that the restricted randomization imposed by the ordering of the runs has negligible effect on the experimental results.

III. CONSTRUCTION OF INTERRUPTABLE DESIGNS

The interruptable designs based on an optimal k+1 run design were developed according to the following procedure. For a k factor experiment an optimal (one which maximizes $|\underline{X}'\underline{X}|$) k+1 run design was used for the initial k+1 guaranteed runs of the interruptable design. Box and Draper (1971) provided such optimal designs for the cases with four, five and six factors, while Mitchell (1972) provided corresponding designs for the eight and nine factor cases. Of course, for seven factors, an eight run 2^{7-4}_{III} fractional factorial (i.e., a Plackett-Burman design) would be optimal and would provide uncorrelated estimates. Therefore, the seven factor case will not be considered further.

Each optimal k+1 run design was augmented by a run to produce a k+2 run design with the maximum possible $|\underline{X}'\underline{X}|$. Dykstra (1971) has shown that this run is equivalent to the point $\underline{x}_0$ in the design space where

$$\mathrm{Var}\ (\hat{y}|\underline{x}_0) = \sigma^2 \underline{x}_0'(\underline{X}'\underline{X})^{-1}\underline{x}_0$$

is maximized. Use was made of this fact in construction of the interruptable designs.

Selection of the k+2$^{\underline{nd}}$ run is not unique because there exists a set of m runs, any of which could be added to the existing design to produce the largest $|\underline{X}'\underline{X}|$. Similar sets of equivalent runs were found when a k+3$^{\underline{rd}}$ or k+4$^{\underline{th}}$ run was to be added. This presented the problem of possible suboptimization. Although all m runs are equivalent for use as the k+2$^{\underline{nd}}$ run, the resulting best k+3 run designs do not necessarily have the same $|\underline{X}'\underline{X}|$.

An example of such suboptimization was found for the eight factor case. The initial design of nine runs was augmented to form a best ten run design. At this stage there were sixteen runs, each of which produced the maximum $|\underline{X}'\underline{X}|$ when included as run eleven. Each of the sixteen possible eleven run designs was formed and the resulting best twelve run design was determined for each case. Four of the sixteen designs led to a best twelve run design with $|\underline{X}'\underline{X}|$ equal to $.2579 \times 10^{10}$, whereas the remaining designs led to twelve run designs with $|\underline{X}'\underline{X}|$ equal to $.2647 \times 10^{10}$. Thus, it is possible to arrive at a suboptimal design depending upon which of the equivalent runs is selected.

To assure that an optimal design is obtained, it is necessary to evaluate designs resulting from each of the equivalent runs at each stage of the design development. This approach was used in defining the best interruptable designs for four, five or six factors. However, as more factors are considered the number of possible designs to evaluate increases dramatically. For example, given the best ten run design for the nine factor case it is necessary to evaluate 502 runs for use as run eleven. Of these, 200 runs qualify as the best eleventh run. For each of the 200 eleven run designs the remaining 501 runs must be evaluated for use as run twelve. Thus a total of 100,702 designs must be considered to generate the best twelve run design when starting from an optimal ten run design. The computer time required for such an extensive evaluation is prohibitive. Therefore, for the eight and nine factor cases only a sample (12%) of the possible best interruptable designs were evaluated. This does not guarantee that the best interruptable design has been determined, although it does provide a lower bound on $|\underline{X}'\underline{X}|$. The resulting design will hopefully be not far from optimal.

The interruptable designs for the alternate strategy were constructed in much the same manner, but the selection of runs was limited to those comprising a Plackett-Burman (P-B) design. That is, a k+1 run design with maximum $|\underline{X}'\underline{X}|$ was selected from the appropriate P-B design. The remaining runs of the P-B design were then added so that the maximum possible $|\underline{X}'\underline{X}|$ was achieved at each stage.

Once again the development of such a design is not unique and the possibility of suboptimization at some design stage must be considered. However, because the runs are restricted to those of a P-B design the number of alternatives to evaluate is greatly reduced. For example, again consider the nine factor case which requires a twelve run P-B design. There are 66 possible ten run designs and twelve possible eleven run designs making a total of 78 designs to evaluate.

With this small number of designs it is possible to verify that the selected interruptable P-B design achieves the maximum possible $|\underline{X}'\underline{X}|$ at each stage. The interruptable P-B design for each of the other cases considered was similarly evaluated to insure the best ordering of the experimental runs.

Representative interruptable $|\underline{X}'\underline{X}|$ designs and P-B designs for four, five, six, eight and nine factors are presented in Figures 2-11. Again it should be noted that these designs are not unique, since other designs with the same determinants were identified. For the $|\underline{X}'\underline{X}|$ designs it is possible that many of these "best" designs are equivalent, that is, identical except for sign changes on some columns and/or permutations of the columns and rows. However, to test if two designs are equivalent it is necessary to

Run No.	x_1	x_2	x_3	x_4
1	1	-1	1	-1
2	1	1	-1	1
3	-1	-1	1	1
4	-1	-1	-1	-1
5	-1	1	1	-1
6	1	1	1	1
7	1	1	-1	-1
8	1	-1	-1	1

Figure 2: Interruptable $|\underline{X}'\underline{X}|$ Design For 4 Factors When 5 Runs Are Guaranteed (First 5 Runs May Be Randomized.)

Run No.	$\underline{x}_1$	$\underline{x}_2$	$\underline{x}_3$	$\underline{x}_4$
1	1	1	1	1
2	1	1	-1	-1
3	1	-1	1	-1
4	-1	1	1	-1
5	1	-1	-1	1
6	-1	1	-1	1
7	-1	-1	1	1
8	-1	-1	-1	-1

Figure 3: Interruptable Plackett-Burman Design For 4 Factors When 5 Runs Are Guaranteed. (First 5 Runs May Be Randomized.)

Run No.	x_1	x_2	x_3	x_4	x_5
1	1	1	1	1	1
2	1	-1	-1	1	-1
3	1	1	1	-1	-1
4	-1	-1	1	-1	1
5	-1	1	1	1	-1
6	-1	1	-1	-1	1
7	1	1	-1	1	1
8	1	-1	1	-1	1

Figure 4: Interruptable $|\underline{X}'\underline{X}|$ Design For 5 Factors When 6 Runs Are Guaranteed (First 6 Runs May Be Randomized.)

Run No.	x_1	x_2	x_3	x_4	x_5
1	1	1	1	1	1
2	1	-1	-1	1	1
3	1	-1	-1	-1	-1
4	-1	1	-1	1	-1
5	-1	-1	1	1	-1
6	-1	1	-1	-1	1
7	1	1	1	-1	-1
8	-1	-1	1	-1	1

Figure 5: Interruptable Plackett-Burman Design For 5 Factors When 6 Runs Are Guaranteed. (First 6 Runs May Be Randomized.)

Run No.	x_1	x_2	x_3	x_4	x_5	x_6
1	-1	1	1	-1	-1	1
2	-1	-1	1	-1	-1	-1
3	-1	1	-1	1	1	-1
4	-1	-1	-1	1	-1	1
5	-1	-1	-1	-1	1	1
6	1	1	-1	-1	-1	-1
7	1	-1	1	1	1	1
8	1	-1	1	-1	1	-1

Figure 6: Interruptable $|\underline{X}'\underline{X}|$ Design For 6 Factors When 7 Runs Are Guaranteed. (First 7 Runs May Be Randomized.)

Run No.	$\underline{x}_1$	$\underline{x}_2$	$\underline{x}_3$	$\underline{x}_4$	$\underline{x}_5$	$\underline{x}_6$
1	1	-1	1	1	1	1
2	1	1	1	-1	1	-1
3	1	1	-1	1	-1	1
4	1	-1	-1	-1	-1	-1
5	-1	1	-1	-1	1	1
6	-1	-1	1	-1	-1	1
7	-1	-1	-1	1	1	-1
8	-1	1	1	1	-1	-1

Figure 7: Interruptable Plackett-Burman Design For 6 Factors When 7 Runs Are Guaranteed. (First 7 Runs May Be Randomized.)

Run No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	1	-1	1	1	1	1	1	-1
2	1	1	-1	1	1	-1	-1	-1
3	-1	1	-1	1	-1	1	1	-1
4	1	-1	-1	-1	-1	-1	1	1
5	-1	1	1	-1	1	-1	1	-1
6	-1	-1	1	1	-1	-1	-1	1
7	-1	-1	-1	-1	1	1	-1	-1
8	1	1	1	-1	-1	1	-1	1
9	-1	1	-1	1	1	1	1	1
10	1	1	1	1	1	1	-1	1
11	1	1	1	1	-1	-1	1	-1
12	1	1	-1	-1	1	-1	1	1

Figure 8: Interruptable $|\underline{X}'\underline{X}|$ Design For 8 Factors When 9 Runs Are Guaranteed. (First 9 Runs May Be Randomized.)

Run No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	1	-1	1	-1	-1	-1	1	1
2	1	1	-1	1	-1	-1	-1	1
3	-1	1	1	-1	1	-1	-1	-1
4	1	-1	1	1	-1	1	-1	-1
5	1	1	-1	1	1	-1	1	-1
6	1	1	1	-1	1	1	-1	1
7	-1	1	1	1	-1	1	1	-1
8	-1	-1	1	1	1	-1	1	1
9	-1	-1	-1	1	1	1	-1	1
10	1	-1	-1	-1	1	1	1	-1
11	-1	1	-1	-1	-1	1	1	1
12	-1	-1	-1	-1	-1	-1	-1	-1

Figure 9: Interruptable Plackett-Burman Design For 8 Factors When 9 Runs Are Guaranteed (First 9 Runs May be Randomized).

Run No.	$\underline{x}_1$	$\underline{x}_2$	$\underline{x}_3$	$\underline{x}_4$	$\underline{x}_5$	$\underline{x}_6$	$\underline{x}_7$	$\underline{x}_8$	$\underline{x}_9$
1	1	-1	1	1	1	1	1	-1	-1
2	1	1	1	1	1	-1	-1	-1	1
3	1	-1	-1	1	-1	-1	-1	1	-1
4	-1	1	-1	1	1	-1	1	-1	-1
5	-1	-1	-1	-1	-1	1	-1	-1	1
6	-1	-1	1	1	-1	-1	1	1	1
7	-1	1	1	-1	1	1	-1	1	-1
8	1	-1	-1	-1	1	-1	1	1	1
9	1	1	1	-1	-1	-1	1	-1	-1
10	1	1	-1	1	-1	1	1	1	1
11	1	1	1	1	1	1	-1	1	1
12	1	1	-1	-1	-1	1	1	1	-1

Figure 10: Interruptable $|X'X|$ Design For 9 Factors When 10 Runs Are Guaranteed. (First 10 Runs May Be Randomized.)

Run No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
1	1	-1	1	-1	-1	-1	1	1	1
2	1	1	-1	1	-1	-1	-1	1	1
3	-1	1	1	-1	1	-1	-1	-1	1
4	1	-1	1	1	-1	1	-1	-1	-1
5	1	1	-1	1	1	-1	1	-1	-1
6	1	1	1	-1	1	1	-1	1	-1
7	-1	1	1	1	-1	1	1	-1	1
8	-1	-1	1	1	1	-1	1	1	-1
9	1	-1	-1	-1	1	1	1	-1	1
10	-1	1	-1	-1	-1	1	1	1	-1
11	-1	-1	-1	1	1	1	-1	1	1
12	-1	-1	-1	-1	-1	-1	-1	-1	-1

Figure 11: Interruptable Plackett-Burman Design For 9 Factors When 10 Runs Are Guaranteed. (First 10 Runs May Be Randomized.)

consider all possible permuations of columns and rows as well as sign changes for the columns. Therefore, such comparisons were not undertaken. It should be noted that even for the relatively small case of five factors and six runs, it is not easy to determine whether two designs are equivalent. For this case, Mitchell pointed out that two designs thought distinct by Box and Draper (1971) are, in fact, equivalent. [See correction to Box and Draper paper.]

IV. EVALUATION OF PERFORMANCE

The $|\underline{X}'\underline{X}|$ criterion which was used in developing the two types of interruptable designs can also be used in evaluating their performance. These determinants along with the maximum known $|\underline{X}'\underline{X}|$ for each case [Mitchell (1972)] are presented in Figure 12. Although these determinants can be used to directly compare designs with the same number of runs, for comparisons between designs of different sizes an alternate criterion is needed.

The ratio of $|\underline{X}'\underline{X}|$ to the maximum $|\underline{X}'\underline{X}|$ of the two interruptable designs was selected as the measure of efficiency for evaluating the designs in the report. These efficiencies are presented in Figure 13. Although the ratio of the $|\underline{X}'\underline{X}|$ to the maximum known $|\underline{X}'\underline{X}|$ could also be used as an efficiency measure, it was not considered here because it is assumed that one of the two interruptable designs must be selected.

From a comparison of the design efficiencies it can be concluded that, as intuition would indicate, $|\underline{X}'\underline{X}|$ designs are superior when the experiment is interrupted very early. However, as more runs are completed the Plackett-Burman designs become the best choice.

No. of Factors	Number of Runs Completed	Interruptable $\|\underline{X}'\underline{X}\|$ Design	Interruptable P-B Design	Design with Maximum Known $\|\underline{X}'\underline{X}\|$
	5	2,304	1,024	2,304
	6	5,120	4,096	5,120
4	7	11,264	12,288	12,288
	8	24,576	32,768	32,768
	6	25,600	16,384	25,600
5	7	61,440	65,536	65,536
	8	147,456	262,144	262,144
6	7	331,776	262,144	331,776
	8	884,736	2,097,152	2,097,152
	9	$.2055 \times 10^9$	$.4777 \times 10^8$	$.2055 \times 10^9$
	10	$.4865 \times 10^9$	$.2866 \times 10^9$	$.6040 \times 10^9$
8	11	$.1148 \times 10^{10}$*	$.1290 \times 10^{10}$	$.1342 \times 10^{10}$
	12	$.2647 \times 10^{10}$*	$.5160 \times 10^{10}$	$.5160 \times 10^{10}$
	10	$.5436 \times 10^{10}$	$.1720 \times 10^{10}$	$.5436 \times 10^{10}$
9	11	$.1208 \times 10^{11}$	$.1032 \times 10^{11}$	$.1288 \times 10^{11}$
	12	$.2684 \times 10^{11}$*	$.6192 \times 10^{11}$	$.6192 \times 10^{11}$

*Lower bound. (Not all cases evaluated.)

Figure 12: Values of $|\underline{X}'\underline{X}|$ for the Two Types of Interruptable Designs and the Design with Maximum Known $|\underline{X}'\underline{X}|$.

No. of Factors	Number of Runs Completed	Interruptable $\lvert \underline{X}'\underline{X} \rvert$ Design	Interruptable P-B Design
4	5	1.0000	.4444
	6	1.0000	.8000
	7	.9167	1.0000
	8	.7500	1.0000
5	6	1.0000	.6400
	7	.9375	1.0000
	8	.5625	1.0000
6	7	1.0000	.7901
	8	.4219	1.0000
8	9	1.0000	.2325
	10	1.0000	.5892
	11	.8901*	1.0000
	12	.5129*	1.0000
9	10	1.0000	.3164
	11	1.0000	.8543
	12	.4336*	1.0000

*Lower bound. (Not all cases evaluated.)

Figure 13: Values of Design Efficiency for Two Types of Interruptable Designs

V. SELECTION OF AN INTERRUPTABLE DESIGN

As emphasized throughout this report, interruptable designs should be considered when the number of runs to be completed is uncertain. Once an experimenter has determined the need for an interruptable design (assuming the designs developed in this report are appropriate for the experimental setting) he or she must select between an $|\underline{X}'\underline{X}|$ or a P-B design. The tables of design efficiencies (Figures 12 and 13) can be used to help arrive at a decision.

If the experimenter has no prior knowledge of when the experiment could be interrupted, one basis for selecting a design is to protect against the worst possible case by adopting a maximin strategy (i.e., by maximizing the minimum possible efficiency). For example, in the four factor case the overall minimum efficiency is .4444 which occurs for the P-B design. Therefore, the $|\underline{X}'\underline{X}|$ design would be used. On the basis of this criterion the $|\underline{X}'\underline{X}|$ design would be used with four, eight or nine factors, while the P-B design would be used with five or six factors.

When prior information relating to the number of runs expected to be completed is available it should be employed in design selection. On the one hand, if it were known with almost certainty that an experiment were going to be interrupted early, the experimenter would no doubt choose the interruptable $|\underline{X}'\underline{X}|$ design. On the other hand, if it were known with almost certainty it would not be interrupted until late (or not interrupted at all), the experimenter's choice would be the P-B design.

The prior information on the expected number of runs in conjunction with the design efficiencies of Figure 13 can be used to calculate the expected efficiency of the two types of designs. These values in turn can be used to choose the appropriate design. For example, consider the four factor case. Suppose the experimenter can estimate the probability that a specific number of runs will be completed. Let p_i be the probability that exactly i runs will be completed. The expected efficiency for the interruptable $|\underline{X}'\underline{X}|$ design is then

$$E(\text{efficiency}) = p_5(1.0) + p_6(1.0) + p_7(.9167) + p_8(.7500). \qquad (1)$$

However, p_8 can be expressed as $1 - p_5 - p_6 - p_7$ so that (1) reduces to

$$E(\text{efficiency}) = .7500 + .2500p_5 + .2500p_6 + .1667p_7 \qquad (2)$$

Similarly, the efficiency for the P-B design can be expressed as

$$E(\text{efficiency}) = 1 - .5556p_5 - .2000p_6 \qquad (3)$$

From equations (2) and (3) it can be shown that the $|\underline{X}'\underline{X}|$ interruptable design has the higher expected efficiency when the following condition is met:

$$.8056p_5 + .4500p_6 + .1667p_7 > .2500$$

For each case considered it is possible to specify the conditions under which the $|\underline{X}'\underline{X}|$ design has the higher expected efficiency. These conditions are summarized in Figure 14. The conditions for the five factor and nine factor cases, which can be illustrated in two dimensions, are indicated in Figures 15 and 16. Once a prior probability structure has been established the inequalities in Figure 14 can be easily applied for selecting the appropriate design.

Number of Factors	Conditions for Selecting the Interruptable $\|\underline{X}'\underline{X}\|$ Design
4	$.8056\ p_5 + .4500\ p_6 + .1667\ p_7 > .2500$
5	$.7975\ p_6 + .3750\ p_7 > .4375$
6	$.7880\ p_7 > .5781$
8	$1.2546\ p_9 + .8979\ p_{10} + .3772\ p_{11} > .4871$
9	$1.2500\ p_{10} + .7121\ p_{11} > .5664$

Figure 14: Conditions Under Which the Interruptable $|\underline{X}'\underline{X}|$ Design Should Be Selected Instead of the Interruptable Plackett-Burman Design

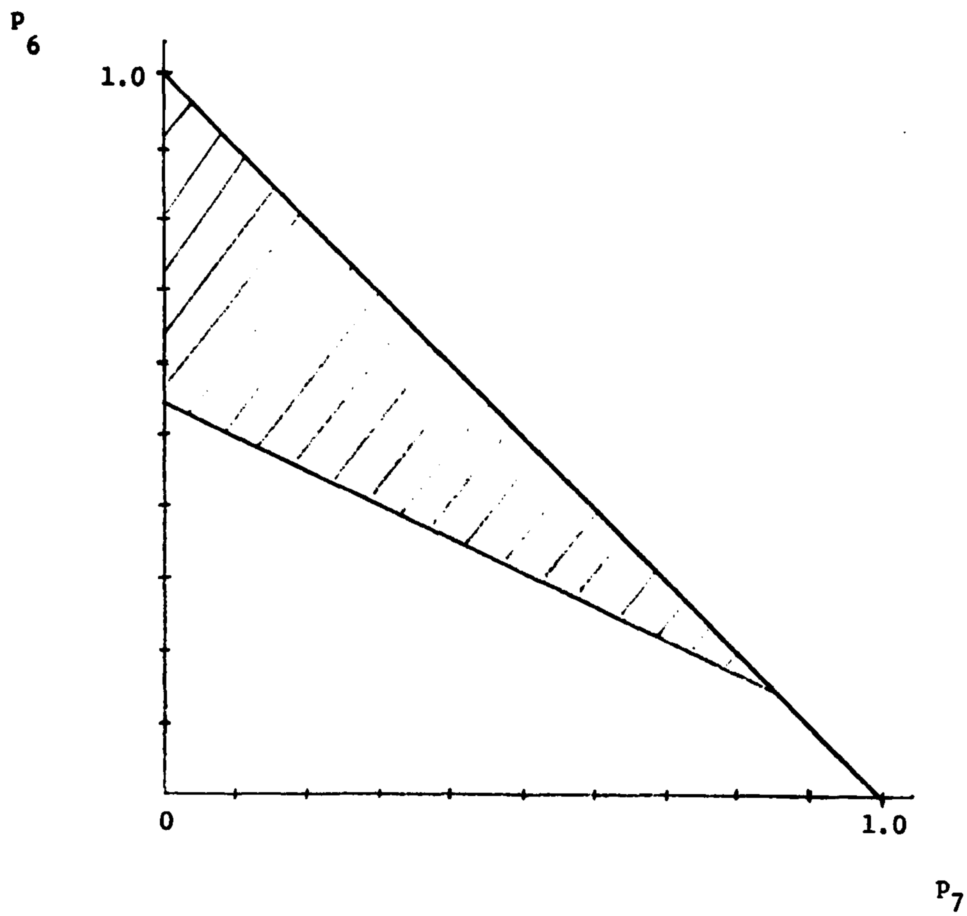

Figure 15: Region Where $|\underline{X}'\underline{X}|$ Design Should Be Used For a Five Factor Experiment

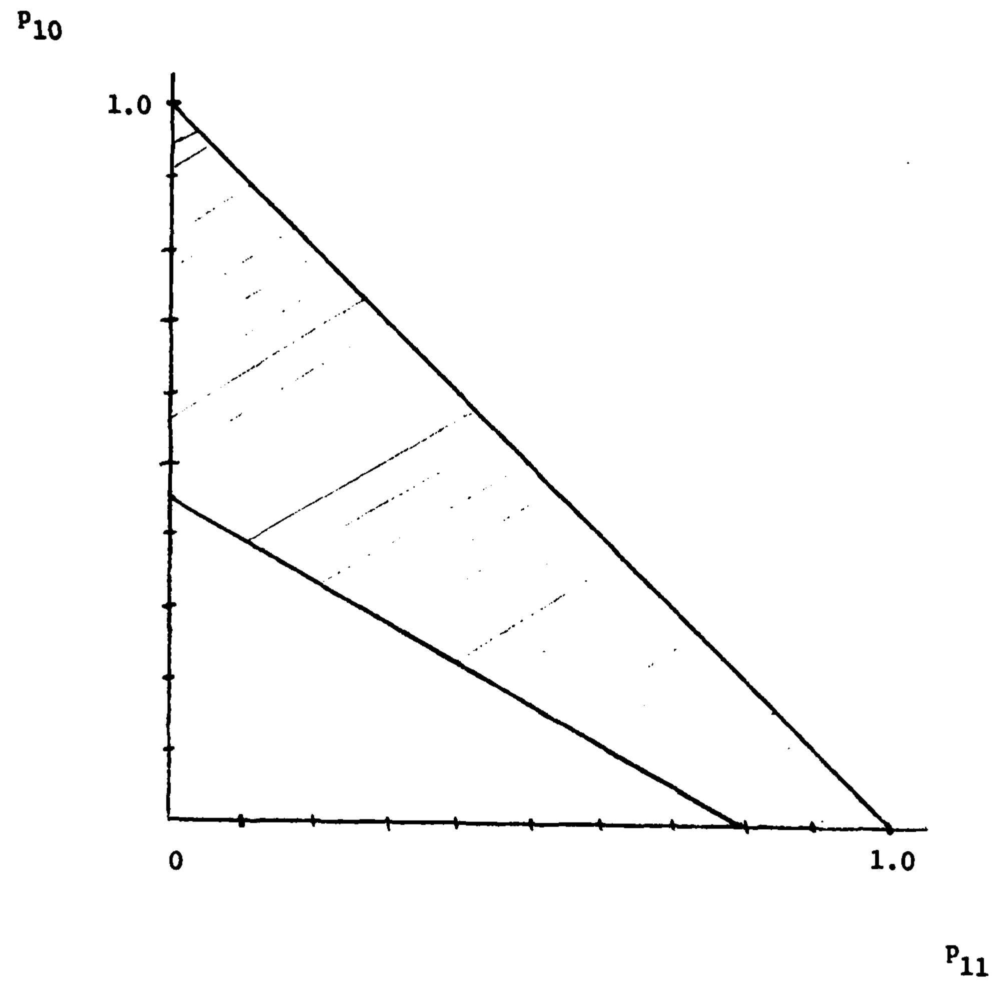

Figure 16: Region Where $|\underline{X}'\underline{X}|$ Design Should Be Used For a Nine Factor Experiment

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Even a well-designed experiment interrupted before its intended completion may prove inefficient, posing serious data analysis problems and leaving important questions unanswered. This report addresses the construction of "interruptable" designs for those cases where a first-order model is assumed and factors at only two levels are considered. The maximum $|\underline{X}'\underline{X}|$ criterion was used in developing "interruptable" designs for two different design strategies which attempt to limit the information lost if an experiment is
(continued--)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) prematurely ended. The resulting designs for four to nine factors are evaluated and selection of an "interruptable" design is discussed.